$X(3872)$ as a $D\bar{D}^*$ MOLECULE BOUND BY QUARK EXCHANGE FORCES

Carlos Peña[1] and David Blaschke[1,2]

[1]Institute for Theoretical Physics, University of Wroclaw, Poland
[2]Bogoliubov Laboratory for Theoretical Physics, JINR Dubna, Russia

The Bethe-Salpeter equation for the T-Matrix of $D^0\bar{D}^{*0}$ scattering is solved with a meson-meson potential that results from 2$^{\text{nd}}$ order Born approximation of quark exchange processes. This potential turns out to be complex and energy dependent due to the pole contribution from the coupling to the intermediate J/ψ - ρ meson pair propagator. As a consequence, a bound state with a mass close to 3.872 GeV occurs in the J/ψ - ρ continuum. This result suggests that quark exchange forces may provide the solution to the puzzling question for the origin of the interaction which leads to a binding of D and $\bar{D}^*$ mesons in the $X(3872)$ state.

1. Introduction

The $X(3872)$ resonance was detected by Belle [1] by examining the invariant mass distribution of particles produced in the decay of B^+ into K^+ π^+ π^- J/ψ. This sighting was later confirmed by BaBar [2]. Even though the particle composition is still under discussion, this resonance is likely to be a $D\bar{D}^*$ bound state with binding energy below 1 MeV [3]. Some attempts have been presented to explain $X(3872)$ based on a T-matrix approach taking into account the neutral and charged D meson channels to properly study the isospin violation in X decay to $J/\psi + \pi^+ + \pi^-$ or $J/\psi + \pi^+ + \pi^- + \pi^0$ [4, 5].

In this contribution we present the derivation of a potential for the $D\bar{D}^*$ interaction which is based on an extension of the separable quark exchange interaction in the $D\bar{D}^* \to J/\psi + \rho$ channel [6] to the 2$^{\text{nd}}$ order Born approximation [7, 8]. An elucidation of the nature of the $X(3872)$ state is important also from the point of view of quark-gluon plasma search since it has been conjectured that in a heavy-ion collision the $c\bar{c}$ state in *statu nacendi* may be subject to important modifications due to its coupling to the $X(3872)$ resonance at the continuum threshold [9, 10]. A generalization [11] of the Matsui model [12] has been applied to the description of the

threshold-like structure in the very precise data from In-In collisions at CERN-NA60 [10]. Its relationship to the theory of the plasma Hamiltonian for heavy quarkonia [13, 14] is currently being worked out [15]. As a partial result of these studies we discuss here the role of quark exchange processes as a possible binding mechanism leading to the $X(3872)$ as a $D\bar{D}^*$ molecule.

2. T-matrix Approach

The Bethe-Salpeter equation for a T-matrix description of $D\bar{D}^*$ scattering is written in the ladder approximation as [8]

$$T(a, a', z) \;=\; U^{(2)}(a, a') + \sum_{a''} U^{(2)}(a, a'')\, G^0_{2D}(a'', z)\, T(a'', a', z) \;, \qquad (1)$$

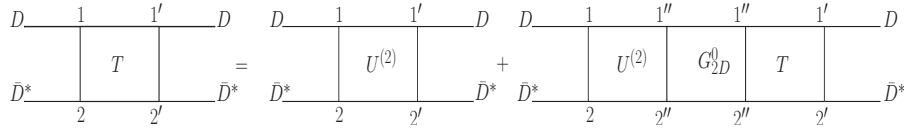which follows from the diagrammatic representation depicted in Fig. 1.

Fig. 1. Diagrammatic representation of Bethe-Salpeter equation for T-matrix of $D\bar{D}^*$ scattering.

For convenience we introduced the shorthand notation $a = \mathbf{p}_1, \mathbf{p}_2$, $a' = \mathbf{p}_{1'}, \mathbf{p}_{2'}$, $a'' = \mathbf{p}_{1''}, \mathbf{p}_{2''}$, $a''' = \mathbf{p}_{1'''}, \mathbf{p}_{2'''}$. Here z refers to the energy of the scattering process described by (1).
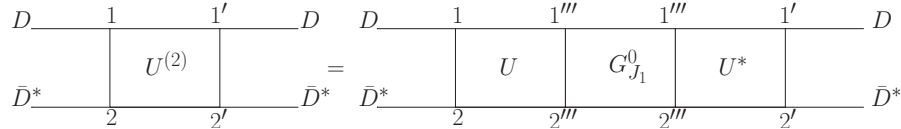

Fig. 2. Diagrammatic representation of the quark exchange interaction kernel for $D\bar{D}^*$ scattering. The subscript J_1 stands for the channel $J/\psi + \rho$.

For the interaction kernel $U^{(2)}$ we suggest here an extension of first Born order diagrams for quark exchange processes in meson-meson scattering [16, 17] to second order (see Fig. 2), where the first order process is given by a separable potential [17, 6]

$$\mathbf{U}(a, a') \;=\; -\lambda\, L(a)\, R(a') \;, \quad \mathbf{U}^*(a', a) = -\lambda\, R(a')\, L(a) \;, \qquad (2)$$

where L and R are the meson form factors, dimensionless functions interpolating between 1 at zero relative momentum and 0 at high momentum. The

amplitude λ of the interaction potential has the dimension of the potential in momentum space, i.e. $[\lambda] = \text{GeV}^{-2}$. The origin of such interaction can be explained by quark exchange forces [8]. With this ansatz the meson-meson potential in the 2nd Born approximation (Fig. 2) leads to a dynamic, separable potential

$$
\begin{aligned}
\mathbf{U}^{(2)}(a, a', z) &= \sum_{a'''} \mathbf{U}(a, a''') \, G^0_{J_1}(a''', z) \, \mathbf{U}^*(a''', a') \\
&= L(a) \, L(a') \, \underbrace{\lambda^2 \sum_{a'''} R^2(a''') \, G^0_{J_1}(a''', z)}_{\mathbf{V}(z)} \\
&= \mathbf{V}(z) \, L(a) \, L(a') \ .
\end{aligned}
\tag{3}
$$

For a separable kernel, the Bethe-Salpeter equation for the T-matrix has a solution in the form

$$
T(a, a', z) = L(a) \, L(a') \, t(P, z) \ ,
\tag{4}
$$

where P is the total conserved momentum of the pair $D\bar{D}^*$. By replacing (2), (3) and (4) into (1) we find the solution

$$
t(P, z) = \frac{\mathbf{V}(z)}{\underbrace{1 - \mathbf{V}(z) \sum_{a''} L^2(a'') \, G^0_{2D}(a'', z)}_{B(P, z)}} \ .
\tag{5}
$$

In the following we use the rest frame with vanishing total momentum ($P = 0$) and the relative momenta p, p', where $\mathbf{p}_{1''} = p$, $\mathbf{p}_{2''} = -p$, $\mathbf{p}_{1'''} = p'$, $\mathbf{p}_{2'''} = -p'$. The function $G^0_{2D}(p, z)$ stands, depending on the considered channel, for one of the nonrelativistic two-particle propagators $G^0_{D_1}(p, z)$ or $G^0_{J_1}(p, z)$, given by

$$
G^0_{D_1}(p, z) = \frac{1}{E_{D_1} - \frac{p^2}{2\mu_{D_1}} \pm i\varepsilon} \ , \quad G^0_{J_1}(p, z) = \frac{1}{E_{J_1} - \frac{p^2}{2\mu_{J_1}} \pm i\varepsilon} \ .
\tag{6}
$$

The abbreviations $D_1 = D^0 + \bar{D}^{*0}$ and $J_1 = J/\psi + \rho$ stand for the two-meson channels and μ_{D_1}, μ_{J_1} for the corresponding reduced masses. In the same framework the binding energies are defined by

$$
E_{D_1} = z - m_{D^0} - m_{\bar{D}^{*0}} \ , \quad E_{J_1} = z - m_{J/\psi} - m_\rho \ .
\tag{7}
$$

We are interested in finding a resonance with a mass just below the threshold of the D_1 continuum, in the region where $m_{J/\psi} + m_\rho \leq z \leq m_{D^0} + m_{\bar{D}^{*0}}$. The potential (3) turns out complex by considering the pole contribution of the two-particle propagator $G^0_{J_1}(p, z)$. This pole located at $p_{J_1} = \sqrt{2\mu_{J_1} E_{J_1}}$

provides the meson-meson potential with sufficient strength to allow the formation of a bound state. Thus integrating around the pole leads to

$$
\begin{aligned}
\mathbf{V}(z) &= \lambda^2 \sum_{a'''} R^2(a''') \, G^0_{J_1}(a''', z) = \frac{\lambda^2 \mu_{J_1}}{\pi^2} \, \lim_{\varepsilon \to 0} \int_0^\infty dp \, \frac{p^2 \, R^2(p)}{p_{J_1}^2 - p^2 \pm i\varepsilon} \\
&= \frac{\lambda^2 \mu_{J_1}}{\pi^2} \int_0^\infty dp \, p^2 \, R^2(p) \left[\frac{\wp}{p_{J_1}^2 - p^2} \mp i\pi \, \delta\left(p_{J_1}^2 - p^2\right) \right] \\
&= \lambda^2 \left[\underbrace{\frac{\mu_{J_1}}{\pi^2} \lim_{\varepsilon \to 0} \int_0^\infty dp \, \frac{p^2 \, R^2(p)(p_{J_1}^2 - p^2)}{(p_{J_1}^2 - p^2)^2 + \varepsilon^2}}_{A_{J_1}(z)} + i \underbrace{\left(\mp \frac{\mu_{J_1} \, p_{J_1} \, R^2(p_{J_1})}{2\pi} \right) \Theta(E_{J_1})}_{B_{J_1}(z)} \right] .
\end{aligned}
\tag{8}
$$

Similarly, the integration around $p_{D_1} = \sqrt{2\mu_{D_1} E_{D_1}}$ gives

$$
\begin{aligned}
B(0, z) &= \mathbf{V}(z) \frac{\mu_{D_1}}{\pi^2} \int_0^\infty dp \, \frac{p^2 \, L^2(p)}{p_{D_1}^2 - p^2 \pm i\varepsilon} \\
&= \mathbf{V}(z) \left[\underbrace{\frac{\mu_{D_1}}{\pi^2} \lim_{\varepsilon \to 0} \int_0^\infty dp \, \frac{p^2 \, L^2(p)(p_{D_1}^2 - p^2)}{(p_{D_1}^2 - p^2)^2 + \varepsilon^2}}_{A_{D_1}(z)} + i \underbrace{\left(\mp \frac{\mu_{D_1} \, p_{D_1} \, L^2(p_{D_1})}{2\pi} \right) \Theta(E_{D_1})}_{B_{D_1}(z)} \right] .
\end{aligned}
\tag{9}
$$

Notice that the step function $\Theta(E_{D_1})$ cancels the term $B_{D_1}(z)$ for $E_{D_1} \leq 0$.

3. Scattering phase shift

The scattering phase shift is determined as

$$
\tan(\delta) = \frac{Im[t(0, z)]}{Re[t(0, z)]} = \frac{B_{J_1}(z) + \left(A_{J_1}^2(z) + B_{J_1}^2(z)\right) B_{D_1}(z)\lambda^2}{A_{J_1}(z) - \left(A_{J_1}^2(z) + B_{J_1}^2(z)\right) A_{D_1}(z)\lambda^2} .
\tag{10}
$$

The expression (10) is very useful to decide whether or not a bound state is located in the region $m_{J/\psi} + m_\rho \leq z \leq m_{D^0} + m_{\bar{D}^{*0}}$. This occurs when $B_{J_1}(z) \neq 0$. Therefore, the analytical continuation of the meson-meson interaction is crucial for explaining resonance formation. An instructive calculation is performed with a Lorentzian form factor (Yamaguchi potential with width parameter γ), using the integral $\int_0^\infty dx \frac{x^2}{(x^2+1)^2(x^2-\tilde{E})} = \frac{\pi}{4} \frac{1}{(1+\sqrt{-\tilde{E}})^2}$ with $\tilde{E} = \frac{2\mu E}{\gamma^2}$. Notice that for $E > 0$ the equality splits into real and imaginary parts, otherwise it is always real [18]. The result is

$$
B_{J_1}(\tilde{E}_J) = \mp \frac{\mu_J \gamma}{2\pi} \frac{\sqrt{\tilde{E}_J}}{(1 + \tilde{E}_J)^2} \theta(\tilde{E}_J) , \quad B_{D_1}(\tilde{E}_{D_1}) = \mp \frac{\mu_{D_1} \gamma}{2\pi} \frac{\sqrt{\tilde{E}_{D_1}}}{(1 + \tilde{E}_{D_1})^2} \theta(\tilde{E}_{D_1}) ,
$$

$$A_{J_1}(\tilde{E}_J) = -\frac{\mu_{J_1}\gamma}{4\pi}\begin{cases} \frac{1-\tilde{E}_J}{(1+\tilde{E}_J)^2} & \tilde{E}_J > 0 \ , \\ \frac{1}{(1+\sqrt{-\tilde{E}_J})^2} & \tilde{E}_J \leq 0 \ , \end{cases}$$

$$A_{D_1}(z) = -\frac{\mu_{D_1}\gamma}{4\pi}\begin{cases} \frac{1-\tilde{E}_{D_1}}{(1+\tilde{E}_{D_1})^2} & \tilde{E}_{D_1} > 0 \ , \\ \frac{1}{(1+\sqrt{-\tilde{E}_{D_1}})^2} & \tilde{E}_{D_1} \leq 0 \ . \end{cases} \tag{11}$$

For a particular choice of λ and γ it is possible that the denominator of the expression (10) vanishes at threshold $z = m_{D^0} + m_{\bar{D}^{*0}} = 3.872$ GeV. The interesting features of these results are shown in Fig. 3 where the scattering phase shift reflects the production of a quasi-bound state by mean interaction of two D-mesons, D^0 and $\bar{D}^{*0}$, passing through the $J/\psi + \rho$ channel.
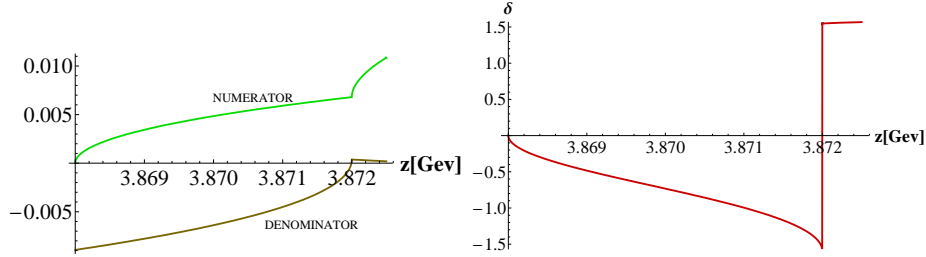

Fig. 3. Plots are done by choosing $\lambda = 20.3$ GeV^{-2}, $\gamma = 0.8$ GeV and sign (+) for imaginary parts. Left Panel: Numerator and denominator of expression (10). Notice the behavior of the denominator just around the threshold of D mesons which vanishes exactly at $z = 3.872$ GeV. Right Panel: The sharp jump of the scattering phase shift by π indicates that a quasi-bound state (resonance) with a mass of $z = 3.872$ GeV appeared. A similar example of such characteristic behavior can be observed in the deuteron channel in nuclear matter: just above the critical temperature for Bose condensation of Cooper pairs a quasi-bound state appears in the continuum, see Fig. 6 of Ref. [19].

4. Conclusions

In this work we have considered the solution of the Bethe-Salpeter equation for the T-matrix of $D^0\bar{D}^{*0}$ scattering which via quark exchange couples to the virtual propagation of a $J/\psi - \rho$ pair. The kernel of the Bethe-Salpeter equation is a complex meson-meson potential obtained by a 2$^{\text{nd}}$ order Born approximation to quark exchange. We have shown that this analytical continuation is crucial for the formation of a bound state in the region $m_{J/\psi} + m_\rho \leq z \leq m_{D^0} + m_{\bar{D}^{*0}}$. The dynamical nature of this complex

potential provides a sufficient enhancement of the strength at the threshold which leads to the $X(3872)$ bound state, rather independent of the detailed dynamics of the model. In a next step, we consider the coupled channel problem, including charged D meson states and the $J/\psi - \omega$ channel [8].

Acknowledgments

REFERENCES

[1] S. K. Choi *et al.* [Belle Collaboration], Phys. Rev. Lett. **91**, 262001 (2003).

[2] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **77** (2008) 111101.

[3] N. Brambilla *et al.*, Eur. Phys. J. C **71** (2011) 1534.

[4] D. Gamermann, J. Nieves, E. Oset and E. Ruiz Arriola, Phys. Rev. D **81** (2010) 014029.

[5] S. Takeuchi, K. Shimizu and M. Takizawa, arXiv:1110.3694 [hep-ph].

[6] K. Martins, D. Blaschke and E. Quack, Phys. Rev. C **51** (1995) 2723.

[7] C. Peña, *Quarkonium production in Heavy Ion Collisions*, PhD Thesis, Wroclaw, in progress (2011).

[8] C. Peña and D. Blaschke, *Binding $D\bar{D}^*$ by quark exchange processes*, in progress.

[9] F. Brazzi, B. Grinstein, F. Piccinini, A. D. Polosa and C. Sabelli, Phys. Rev. D **84**, 014003 (2011).

[10] D. Blaschke and C. Peña, Nucl. Phys. Proc. Suppl. **214**, 137 (2011).

[11] C. Peña and D. Blaschke, arXiv:1108.4180 [nucl-th].

[12] T. Matsui, Annals Phys. **196**, 182 (1989).

[13] D. Blaschke, arXiv:0912.4479 [hep-ph].

[14] R. Rapp, D. Blaschke and P. Crochet, Prog. Part. Nucl. Phys. **65**, 209 (2010).

[15] D. Blaschke and C.Peña, *Contribution of X(3872) to the plasma Hamiltonian for charmonium in hot dense matter*, in progress.

[16] T. Barnes and E. S. Swanson, Phys. Rev. D **46**, 131 (1992).

[17] D. Blaschke and G. Röpke, Phys. Lett. B **299**, 332 (1993).

[18] G. Röpke, M. Winkel, *Green's function technique for statistical ensembles*, Institut für Physik, Rostock University (2009).

[19] M. Schmidt, G. Röpke and H. Schulz, Annals Phys. **202**, 57 (1990).